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Finite Element Model Updating Using Wavelet Data and Genetic Algorithm

Tshilidzi Marwala*
Imperial College,
London, England SW7 2BT, United Kingdom

I. Introduction

FINITE element models are widely used to predict the dynamic characteristics of aerospace structures. These models often give results that differ from the measured results and therefore need to be updated to match the measured data. Some of the updating techniques that have been proposed to date use the time, modal, frequency, and time-frequency domain data.[1] In this study a finite element model updating method that uses wavelet data and genetic algorithm is used to update the finite element model of a beam. The main advantage of using wavelet data is that they reveal information in both the time and frequency domains.[2] The shortcoming of using the wavelet data is that there is a great deal of information, that is, frequencies and their evolution through the time. This increases the presence of local optimum solution when the distance between the measured and predicted spectra is minimized.

The proposed wavelet finite element updating technique is compared to conventional technique that uses frequency response functions (FRFs)[1] using results obtained from measurements taken on a suspended beam. Genetic algorithm (GA)[3] is used to minimize the distance between the wavelet data predicted from the finite element model and those from the measured data. Similarly, GA is used to minimize the distance between the FRFs predicted from the finite element model and those calculated from the measured data. GA is a probabilistic search algorithm that is capable of finding globally optimum results, and its implementation consists of crossover, mutation, and reproduction.[3]

II. Mathematical Foundation

A. Wavelet Data

Wavelet transform (WT) of a signal is an illustration of a timescale decomposition, which highlights local features of a signal. Wavelets occur in sets of functions, that are defined by dilation that controls the scaling parameter, and translation, which controls the position of a single function known as the mother wavelet. Wavelet transform W of a signal $X(\omega)$ in frequency domain can be written as[2]

$$W(j,k) = \int_{2\pi 2^j}^{4\pi 2^j} X(\omega) e^{i\omega k/2^j}\, \mathrm{d}\omega \tag{1}$$

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*Postdoctoral Research Associate, Department of Electrical and Electronic Engineering, Exhibition Road; currently [**job title**], P.O. Box 787391, Sandton, 2146, South Africa; tmarwala@yahoo.com.

In Eq. (1) ω is the frequency, and k determines the position of the wavelet and gives an indication of time. The parameter j is called the level and determines how many wavelets are needed to cover the mother wavelet and is the same as the frequency varying in harmonics. In this Note the expression $X(\omega)$ in Eq. (1) is substituted by the FRFs represented in the modal domain using the modal summation equation given in Ref. 4. Using the FRFs ensures that the wavelet data are independent of the excitation on the structure. Equation (1) can therefore be written as follows[5]:

$$W_{\mathrm{pl}}(j,k) = \int_{2\pi 2^j}^{4\pi 2^j} \sum_{n=1}^{N} \frac{-\omega^2 \phi_p^n \phi_l^n}{-\omega^2 + 2\zeta_n \omega_n \omega i + \omega_n^2} e^{i\omega k/2^j}\, \mathrm{d}\omega \tag{2}$$

In Eq. (2) W is the wavelet caused by the excitation at p and measurement at l, ω_n is the nth natural frequency, N is the number of modes, ϕ^n is the nth mode shape, $i = \sqrt{-1}$, ω is the frequency point, and ζ_n is the nth damping ratio. Equation (2) shows that the wavelet of a signal can be expressed in terms of the modal properties.

B. Finite Element Updating

The parameters that are updated are selected using engineering judgment. Because it is assumed that the discrepancy between the measured data and the finite element model is as a result of erroneous moduli of elasticity of the structure, the modulus elasticity of each element in the finite element model is treated as a design variable. An objective function, which describes the function to be minimized, is formulated by calculating the distance between the wavelet data from the finite element model and from the measurements and can be written as:

$$E = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{l=1}^{L} \sum_{p=1}^{P} \left\| \log \left\| W_{\mathrm{pl}}^{m}(j,k) \right\| - \log \left\| W_{\mathrm{pl}}^{a}(j,k) \right\| \right\|^2 + \gamma \sum_{i=1}^{N} x_i \tag{3}$$

where W is the wavelet data, $\| \|$ is the absolute value, K is the maximum representation of time, J is the maximum representation of frequency, L is the number of measurement points, P is the number of excitation points, m is measured, and a is analytical (from finite element model). The second term in Eq. (3) is the regularizer,[1] x_i is the ith modulus of elasticity, γ is the parameter that ensures that the regularizer does not dominate the cost function, and N is the number of design variables. Using genetic algorithm, the moduli of elasticity of every element are updated, thereby modifying the finite element model. Alternatively, the distance between the measured and analytical FRFs can be written as follows:

$$E = \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{k=1}^{K} \left\| \log \left\| FRF_{\mathrm{kl}}^{m}(\omega_i) \right\| - \log \left\| FRF_{\mathrm{kl}}^{a}(\omega_i) \right\| \right\|^2 + \gamma \sum_{i=1}^{N} x_i \tag{4}$$

The parameter γ is chosen such that the value of the first term of Eqs. (3) and (4) are 10 times the value of the second term (the regularizer) of these equations. These criteria were arrived at through trial and error.

C. Genetic Algorithms

Genetic algorithms were inspired by Darwin's theory of natural evolution. This natural optimization method is used to optimize the cost function in Eqs. (3) and (4). The genetic algorithm implemented was suggested by Holland[6] and acts on a population of binary-string chromosomes. Each of these strings is the discretized representation of a point in the search space and therefore has a fitness function given by the objective function. On generating a new population, three operators are performed: 1) crossover, 2) mutation, 3) and reproduction. The crossover operator mixes genetic information in the population by cutting pairs of chromosomes at random points along their length and exchanging over the cut sections. This has a potential of joining successful operators together. The arithmetic crossover technique[3] is used in this Note. The mutation operator picks a binary digit of the chromosomes at random and inverts it. This has a potential of introducing to the population new information. Reproduction takes successful chromosomes and reproduces them in accordance to their fitness functions. On implementing the proposed updating technique, the initial population of updated models is estimated. In this Note the size of the population and the number of parameters are both 10. In this study the population size is kept at 10 by deleting the most unfit "children" during the reproduction cycle. The current parameters to be updated are then transformed from floating point to 16-bit format using Gray coding.[3] The chromosomes (individuals in the population) represented by binary numbers are allowed to interact by using one of the three crossover procedures just mentioned. The probability of crossover occurring is set to 0.6. Each chromosome mutates at a probability of 0.0333. The chromosomes are transformed back to floating point parameters. The objective functions given by Eqs. (3) and (4) are then used to evaluate the fitness of each parameter. The parameters that are fit are allowed to reproduce, and the weak parameters are eliminated using the roulette wheel[3] procedure. If the model in the population has converged, then the procedure is terminated. Otherwise the parameters are transformed back to binary form, and crossover, mutation, and reproduction procedures are repeated.

III. Example: Experimentally Measured Beam

To test the proposed procedure, a freely suspended steel beam is used. The beam has the following dimensions: length, 1.0 m; width, 39.0 mm; thickness, 5.93 mm. The beam is exited at 10 equidistant positions, using the modal hammer with a sensitivity of 4 pC/N and head of mass 6.6 g and a cutoff frequency of 3.64 kHz. Acceleration is measured at a fixed position with an accelerometer of a sensitivity of 2.6 pC/ms^{-2} and a mass of 19.8 g. The details of this beam are found in Ref. 7, and the finite element model with 10 elements is constructed. The beam is excited at various positions using modal hammer, and acceleration is measured at a fixed position. Using conventional signal processing analysis, the measured data are transformed into the FRFs and wavelet data. Using these data, finite element model updating is performed.

IV. Discussion

The sample wavelet results for the experimental measurements and initial and updated finite element models are shown in Fig. 1. When the natural frequencies from the updated models are compared to those calculated from the initial model as well as from measured data, the results in Table 1 are obtained. The number of generations required to reach convergence are 15 and 13, respectively, for the FRF- and wavelet-based updating method. This table shows that for modes 1, 2, 6, and 7 the wavelet data give a better-updated model than the FRF data, whereas for modes 3, 4, and 5 the reverse is the case. For all of the modes, the updated model is better than the initial model. To compare the analytical mode shapes to the measured mode shapes, the modal assurance criterion (MAC) is used.[8] The MAC is a criterion that represents how two mode shapes are correlated. Two perfectly correlated mode shapes give an identity matrix. As a result, in this Note the diagonal of the MAC whose elements are supposed to be equal to one for similar mode shapes is used to assess the effectiveness of the proposed updating method. The diagonal of the MAC between the experiment and various updated models are shown in Table 2. This table shows that, on average, the wavelet-updated model gives marginally more accurate mode shapes than the FRF-updated model. However, the updated models are only nominally better than the initial model.

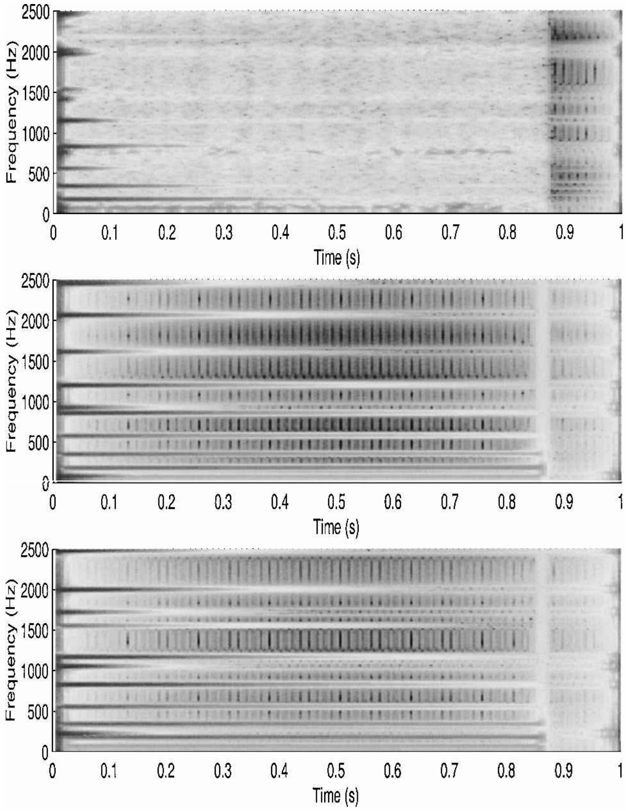

Fig. 1 From top to bottom: experimental, initial, and updated wavelet data.

Table 1 Measured natural frequencies and those calculated from an initial finite element model and updated finite element models

Mode number	Experiment, Hz	Initial FEM,[a] Hz	FRF, Hz	Wavelet, Hz
1	30.7	31.6	30.9	30.4
2	84.6	87.2	85.0	84.6
3	166.1	171.0	166.1	165.7
4	275.6	283.2	274.1	271.8
5	411.6	424.1	412.2	407.9
6	571.7	594.9	582.7	576.3
7	763.9	796.5	783.2	772.3

[a]Finite element model.

Table 2 Modal assurance criterion between the measured mode shapes and various finite element calculated mode shapes

Mode number	MAC experiment/initial	MAC experiment/FRF	MAC experiment/wave
1	0.9961	0.9965	0.9958
2	0.9895	0.9885	0.9899
3	0.9799	0.9785	0.9791
4	0.9703	0.9698	0.9696
5	0.9712	0.9692	0.9710
6	0.9483	0.9483	0.9497
7	0.8940	0.8952	0.8914

V. Conclusions

In this Note an updating procedure, which uses wavelet data, is compared to that which uses FRF data. Genetic algorithm was used to optimize the cost functions between the measured and analytical data. The advantage of using genetic algorithm is that it is able to identify globally optimum results. It is found that, on average, the wavelet-updated model performs better than the FRF-updated model.

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Aeroelastic Response of an Airfoil-Aileron Combination with Freeplay in Aileron Hinge

H. Alighanbari*

Ryerson University, Toronto, Ontario M5B 2K3, Canada

Introduction

RECENTLY, increasing attention has been devoted to nonlinear aspects of aeroelastic problems. One particular nonlinearity that has received considerable attention is bilinear structural stiffness. This is a good representation of either loose or worn control-surface hinges. Freeplay nonlinearity was first considered by Woolston et al.[1] Their results, both analog simulation and experimental, showed that depending on the initial pitch displacement limit-cycle oscillations (LCOs) can occur for velocities significantly less than the linear flutter velocity. Flutter analyses of airfoils with freeplay structural nonlinearities were then continued by several researchers, for example, Breitbach,[2] McIntosh et al.,[3] Tang and Dowell,[4] and Price et al.[5] These studies confirmed the existence of complicated dynamics well below the linear flutter velocity. Recently, Kholodar et al.[6] analyzed the behavior of a three-degree-of-freedom (DOF) airfoil via numerical time integration using a standard state-space approximation to Theodorsen aerodynamics, and they obtained a variety of nonlinear behaviors.

The objective of this Note is to show some interesting dynamical behaviors of a three-DOF airfoil-aileron combination subjected to incompressible airflow taking into account a freeplay structural nonlinearity in the aileron hinge moment. At first, because the airfoil motion can be nonperiodic, aerodynamic forces for arbitrary motions of a three-DOF airfoil are derived from Theodorsen's equations using Laplace transformation.[7] Then, the airfoil's dynamical behavior and possibility of nonperiodic motions are investigated using both a finite difference method and a dual-input describing function technique.[5] Results from both methods are presented here because the finite difference results do not show complete continuous form of the LCOs, and the describing function method is not able to give nonperiodic solutions.

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*Assistant Professor. Department of Mechanical, Aerospace and Industrial Engineering. Member AIAA.

Aeroelastic Equations

The equations of motion for the two-dimensional airfoil-aileron combination shown schematically in Fig. 1a can be written in nondimensional form as

$$\xi''(\tau) + x_\alpha\alpha''(\tau) + x_\beta\beta''(\tau) + 2\zeta_\xi(\bar{\omega}_\xi/U)\xi'(\tau) + (\bar{\omega}_\xi/U)^2\xi = p(\tau,\xi,\alpha,\beta) \tag{1}$$

$$\left(x_\alpha/r_\alpha^2\right)\xi''(\tau) + \alpha''(\tau) + \left(z_\beta/r_\alpha^2\right)\beta''(\tau) + 2\zeta_\alpha(1/U)\alpha'(\tau) + (1/U^2)\alpha = r(\tau,\xi,\alpha,\beta) \tag{2}$$

$$\left(x_\beta/r_\beta^2\right)\xi''(\tau) + \left(z_\beta/r_\beta^2\right)\alpha''(\tau) + \beta''(\tau) + 2\zeta_\beta(\bar{\omega}_\beta/U)\beta'(\tau) + (\bar{\omega}_\beta/U)^2H(\beta) = w(\tau,\xi,\alpha,\beta) \tag{3}$$

where $\xi = h/b$ is the nondimensional heave displacement; $(\,)'$ denotes differentiation with respect to nondimensional time $\tau = tV/b$; $H(\beta)$ is a nonlinear function representing the restoring moment in the aileron hinge normalized with respect to the linear stiffness; p, r, and w are the nondimensional aerodynamic force and moments defined as $p(\tau) = -L/(mV^2/b)$, $r(\tau) = -M_\alpha/mV^2r_\alpha^2$, and $w(\tau) = -M_\beta/mV^2r_\beta^2$; U is nondimensional airspeed; r_α is the nondimensional airfoil radius of gyration about the elastic axis, r_β is the nondimensional aileron radius of gyration about the aileron hinge line, and $z_\beta = r_\beta^2 + (c_\beta - a_h)x_\beta$; and ζ_ξ, ζ_α, and ζ_β are viscous damping ratios in plunge pitch and aileron, respectively. $\bar{\omega}_\xi$ and $\bar{\omega}_\beta$ are uncoupled frequency ratios defined as $\bar{\omega}_\xi = \sqrt{[(K_\xi/m)/(K_\alpha/I_\alpha)]}$ and $\bar{\omega}_\beta = \sqrt{[(K_\beta/m)/(K_\alpha/I_\alpha)]}$, where m is the mass of airfoil-aileron; I_α the mass moment of inertia of the airfoil-aileron about the elastic axis; I_β the mass moment of inertia of the aileron about the aileron hinge; and K_ξ, K_α, and K_β are linearized stiffnesses in plunge, pitch, and aileron hinge, respectively.

Because of the possibility of nonperiodic motions of the airfoil, Theodorsen's equations cannot be employed in the present analysis. Thus, the aerodynamic force and moments are derived for any

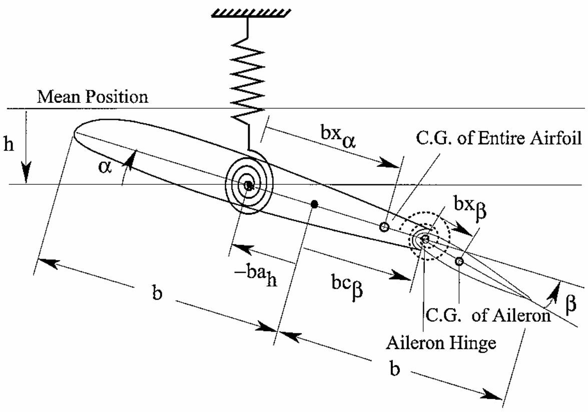

Fig. 1a Schematic of the three-DOF airfoil.

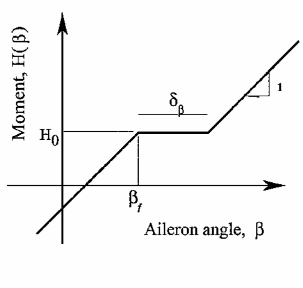

Fig. 1b Schematic of the freeplay nonlinearity.